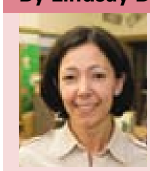
Ten Is the Magic Number!

How this teacher develops composition of ten with second graders was dramatically reshaped by the 2006 release of NCTM's Curriculum Focal Points.

By Lindsay Barker



Lindsay Barker, lbarker@ric.edu, teaches second grade at the Henry Barnard Laboratory School on the Rhode Island College campus in Providence. She is interested in the development of number sense and is involved in curriculum development and evaluation at the school.

Bernard, an inquisitive and friendly second grader, noticed an adult visitor in my classroom observing another child. Bernard loves to chat with adults, to share his wealth of information about the world, and to quiz grownups on his latest fascinating number puzzle. This day, Bernard had a math question he just had to share with the visitor. "What is 14,000 minus 99?" he asked with sincere wonder. We had spent the first five weeks of school studying patterns in numbers and learning addition strategies for basic facts. We had not touched on subtraction, let alone large numbers—hence Bernard's fascination.

My toes curled at the visitor's reply: "You need to line up the digits."

This is not the reply I want a curious second grader to hear to prompt his thinking. Bernard had no idea what "line up the digits" meant, and he was trying to do the problem mentally. So, what do I wish the adult visitor had said to my student? I wish she had replied to his question with one of her own.

Reshaping How I Teach

The suggestion that number sense and computation be focal points of the second-grade mathematics curriculum (NCTM 2006) resulted in positive changes in my approach to teaching these areas. Receiving extended instructional time on the composition and decomposition of ten and its applications allowed my students to become more efficient at computation and develop greater number sense and had a positive affective impact on the way they view mathematics.

I called Bernard over as soon as I could. "What do you know about 99?" I asked him.

It's almost 100," he replied. "Just like a nine is almost a ten."

"What happens if you imagine the 99 is 100 and you take away 100 [from 14,000]?" I asked.

Bernard has good number sense. He knew that nine hundreds would be left from one thousand and quickly got to the answer of 13,900: "Oh, yeah. You need to remember the one. So, it is 13,901!" he exclaimed.

"BINGO!" I rejoiced.

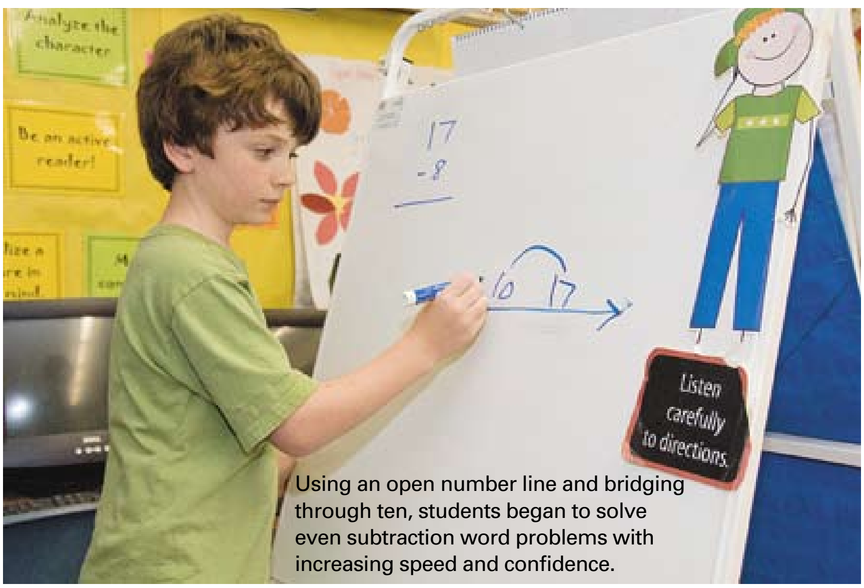
The Focal Points and Curriculum Development

Bernard's is the kind of thinking that I am determined to help all of my students develop. *Curriculum Focal Points* suggests three main areas on which to focus the mathematics curriculum at each grade level. The book calls for the second-grade curriculum to focus on using "understanding of addition to develop quick recall of basic addition and related subtraction facts" (NCTM 2006). *Curriculum Focal Points* also calls for efficient methods for calculating sums and differences mentally and for fluency with efficient procedures.

The common criticism of U.S. mathematics curricula as a mile wide and an inch deep had sometimes felt close to the truth whenever I had tried to move my students along a diverse curriculum path after teaching them some strategies to solve basic addition and subtraction problems. I saw the release of *Curriculum Focal Points* as a positive step toward curriculum evaluation and development. It affirmed my thought that I should be devoting more time in my second-grade classes to developing number sense and efficient computation strategies so that all my students had the opportunity to develop the level of number sense that Bernard had.

I am in the fortunate position of working in a laboratory school, a function of which is curriculum development. As a member of the school mathematics team, one of my roles is to organize data collection about teaching mathematics at each grade level and form a scope and sequence document for the school. This document is evaluated as we determine the effectiveness of materials and how and what concepts are taught each year. The timing of this project coincided with the release of *Curriculum Focal Points*. I made a personal decision to evaluate my second-grade curriculum in light of the Focal Points and take a more in-depth approach to developing number sense and computation—in contrast to the skill-drill fashion that the media supported at the time *Curriculum Focal Points* was released. I wanted a more coherent approach with greater application and connections. My team teacher also felt strongly about developing these areas during the early childhood years and was very willing to alter the pace and depth of our scope and sequence.

After refocusing my second-grade curriculum to recognize the importance of number sense in computation and to allow more time for in-depth study of these areas, the immediate effects were an increase in mathematical dialogue and unexpected affective changes, such as students' feelings of greater efficiency and confidence with number relations. If this in-depth approach proved to provide my students with a more solid foundation for third grade, the third-grade teachers also might be willing to evaluate their curriculum in light of the Focal Points. The process could have a positive domino effect through our entire school.



Using an open number line and bridging through ten, students began to solve even subtraction word problems with increasing speed and confidence.

Composing Ten, Factual Fluency, and Dialogue

Because our U.S. number system is based on ten, I decided that developing composition of ten would be the backbone of my curriculum. One of the key manipulatives I used is the tens frame (see **fig. 1**), which is basically two rows of five squares filled with black dots to represent the number you are showing (Van de Walle 2007).

Thompson and Van de Walle (1984) wrote about the importance of number combinations that total ten, crediting Robert Wirtz of Curriculum Development Associates with introducing them to the idea of the tens frame. I spent a week acquainting my students with tens frames as a visual reference for how ten is composed. They looked at the frames and initially noticed that ten is two rows of five. They then began to discuss each number in terms of its relationship to ten or to five. Seven became three less than ten. We flashed the tens frames in quick-fire rounds, with students identifying both the number of dots and how many more dots would be needed to make ten. In this way, my students became proficient in composing ten. They played Tens Go Fish and Turn over Ten from the Investigations in Number Data and Space series (Economopoulos et al. 1998). Each game involves students finding number pairs that add up to ten. Subsequently, students used completed tens frames to practice solving problems with an addend of ten (e.g., 10 + 7), which nicely highlighted the place-value aspect of numbers in the teens.

Our next step was to move to addition questions that involve nine as an addend. Students had found that addition with an addend of ten is a simple mental computation, so they began using the tens frame to take one from one addend and give it to the nine to compose a ten (9 + 8 became 10 + 7; see **fig. 2**). We not only modeled addition with tens frames and counters but also proved equivalence with a math balance from which we hung weights with numbers on them.

Students also became proficient at turning eights and sevens into tens. We used tens frames to model both addends, and we shifted counters from the lower addend to fill the tens frame of the higher addend. This visual aid helped students understand that removing parts from one addend and adding them to another does not change the total quantity; the balance scale beautifully represents this concept. Making tens made sense to the students, and soon they began doing it mentally.

The development of number sense in the early grades is crucial to computational fluency and understanding in later grades. I have realized the importance of providing students with the time and opportunities to compose and decompose numbers, to discover relationships between numbers, and to find alternative ways of describing them: "When children are forced to work with greater numbers before they can work fluently with numbers to ten, they become dependent upon rules and procedures that have no meaning" (Postlewait, Adams, and Shih 2003). This dependence on rules had been my educational experience; my mental mathematics skills had never developed in the way that Bernard's have. I wanted to allow all my students the time to compose and decompose numbers and develop a sense of the relationship between them.

Once students were confident with bridging to ten, it was time to extend the concept to numbers up to twenty and then beyond. We progressed quickly, beginning by modeling numbers in the teens and describing them as a tens group and some singles. Because the tens frame clearly demonstrates how far away from the next ten a number is, the relationship between numbers was a key focus of our discussions. Earlier practice at quickly determining how many more counters were needed to fill the tens frame had provided the foundation for this work. Other models I have used in the past have not been as successful in demonstrating this key relationship because the ones must be continually counted, and the distance to the next ten is not

Figure 1

The tens frame is a key manipulative for developing composition of ten.

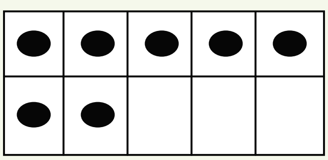
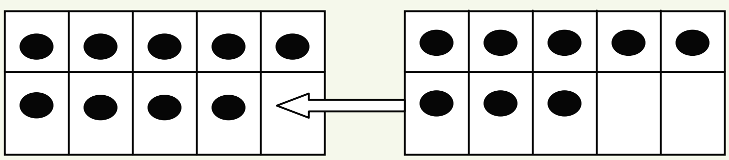


Figure 2

Students began to use the tens frame when they had an addend of nine.



visually displayed. With the tens frame, however, students excitedly connected the higher numbers to the concept of groups of ten.

Focusing on groups of ten naturally brought place-value discussions into our lessons. We had an interesting discussion about the Chinese number system, in which numbers are named by their relationship to ten. Twelve is "ten-two," and thirty-four is "three tens-four." Some researchers suggest that this naming convention is why Chinese students naturally have a better understanding of number than their counterparts in other cultures do. My students eagerly linked our tens-frame models to the Chinese number system and wondered why our naming system does not make as much sense.

The next day, when I asked, "Thirteen plus what makes twenty?" Lucille was eager to share her strategy: "Well, I know that thirteen is a teen. It has a group of ten and three ones. Twenty is two tens, and you have one ten already in the thirteen. We all know that three plus seven is ten, so you just need seven more to make twenty." Other students nodded in agreement.

To reinforce the composition of twenty, we played Cover Up from the Investigations series. One student showed a whole set of twenty cubes, and then covered up part of the set with a folder. The other student could see the remaining part and had to figure out what was covered. To solve the problem, students applied their knowledge of making groups of ten. The paper-and-pencil task that followed (Van de Walle 2007) encouraged them to quickly identify two (of three) numbers that have a sum of twenty. For example, of the three numbers thirteen, six, and seven, they were to find the pair that equals twenty when added.

When my students were confident in composing tens mentally, it was time to move to subtraction. In the past, I have struggled to get students proficient in subtraction. Some students never progress from relying on counting backward in ones. I wanted to build on the idea of tens because it had made sense to my students, and their mental calculations were developing well. Their success prompted me to introduce them to the open number line. (O'Laughlin described her use of this tool in *Teaching Children Mathematics* [2007]). The open number line is simply a line with no prewritten numbers on it. Students determine the start and finish number on the basis of numbers in the question they are solving and then use "friendly numbers" (in our case, multiples of ten) to jump from the start number to the finish number. Then they calculate how far they jumped.

We began our lessons by setting a context for finding a difference. Two students connected Unifix® cubes to show the length of their feet. One student's foot length was eight cubes; the other's was fourteen. To review addition strategies, we discussed combined shoe length. Again students showed their flexibility with number as they took six from the eight to give to the fourteen to make two groups of ten, and then added the final two to get a total of

Figure 3

Ten is the magic number! Students became confident in mentally composing tens and progressed to using an open number line for subtraction.

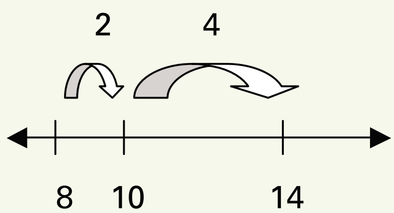
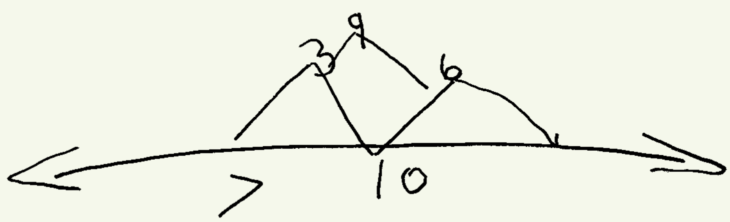


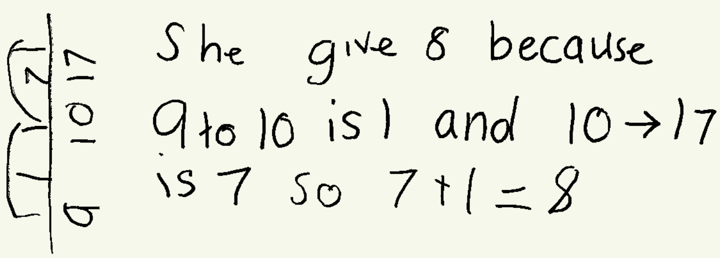
Figure 4

Using an open number line and bridging through ten, students began to solve word problems with increasing speed and confidence.

(a)
Julia was playing Cover Up with Haley. They had 16 cubes altogether. Haley covered some up, and Julia could see only 7. How many did Haley cover up?



(b)
I grew 9 pumpkins in my garden. A friend gave me some of hers, and then I had 17 altogether. How many did she give me? Show your work and explain your thinking in words.



twenty-two. When I asked, “What is the *difference* between the lengths of the shoes?” one student put the towers of cubes side by side and compared them. She counted the difference in ones. Another student counted down by ones.

Then Harry exclaimed, “Just use the tens strategy! Eight plus two more is ten, and four more makes six altogether!” (See **fig. 3.**) Students began to use the open number line and bridge through ten to answer questions (see **fig. 4**). My second graders were solving problems involving differences with a speed and confidence that was significantly better than in previous years when I had not taken the time to allow such an in-depth study of tens.

During one of our discussions about the open number line, Hazel wondered aloud, “It can be used for bigger numbers too, right?” I immediately wrote a double-digit problem on the white board: 27 + ? = 53. All the students began talking and asking to come to the board to solve it. I called Hazel up. “Hazel, how could knowing about tens help you [solve this problem]?”

She proceeded to put 27 on the left of her number line and 53 on the right, and then paused. After thinking for a few minutes, she added 30 close to the 27 (see **fig. 5**): “It takes three to get to thirty, then twenty to get to fifty, and another three. That’s twenty-six!”

Another student added, “You can go back the other way, too. It takes three to get back to fifty, then twenty to get down to thirty, and then three more to get down to twenty-seven. That’s twenty-six altogether.”

Figure 5

When Hazel wondered if the open number line could be used for larger numbers, the teacher immediately challenged her with a double-digit problem, which Hazel successfully solved.

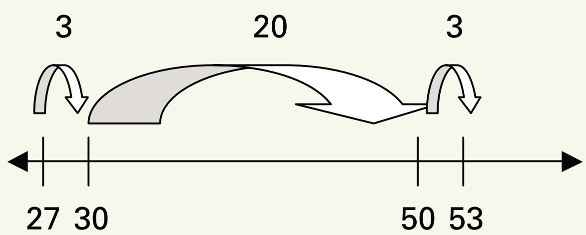


Figure 6

Stan's responses to the questions on the end-of-the-year assessment affirmed his teacher's earlier decision to study tens in depth.

19) Do you think that you have improved in math so far in second grade? Tell me about your improvement.

I youst to have too
count up now I just
know the anse

20) Have your feelings about math changed at all this year? Tell me about them.

yes I find it out
faster I Dont fell
like this will take
me a wile.

Figure 7

During the year, Victoria's confidence soared, and she recognized the change in herself.

19) Do you think that you have improved in math so far in second grade? Tell me about your improvement.

I think improved with yousing
tens

20) Have your feelings about math changed at all this year? Tell me about them.

My feelings changed and I feel more confedent
about math.

Figure 8

Lucille now loves math!

20) Have your feelings about math changed at all this year? Tell me about them.

Last
year I dident like math
that mutch but nowe I love math

Affective Changes

In addition to greater reasoning and factual fluency, I have noticed an increase in my students' overall confidence with mathematics. They are eager to share and excited when they apply bridging strategies to larger numbers. To find out more about the affective results of this approach, I included the following two questions at the end of students' assessments:

1. Do you think that you have improved in math so far in second grade? Tell me about your improvement.
2. Have your feelings about math changed at all this year? Tell me about them.

The responses were an affirmation of the work we had done (see **figs**. **6**, **7,** and **8**). Van de Walle writes of this affective change, "Relational understanding has an affective effect as well as a cognitive effect. When ideas are well understood and make sense, the learner tends to develop a positive self concept about his or her ability to learn and understand mathematics…. There is no reason to fear or be in awe of knowledge learned rationally" (2007, p. 27).

An outcome of refocusing my curriculum that I had not anticipated, this affective change is the one that I value the most.

Teachers' Habits of Mind

Teachers must possess the correct habit of mind about mathematics. Griffin (2004) believes that teachers must see mathematics as a set of conceptual relationships between numbers and number symbols rather than as numbers that are manipulated by rules. The questions we ask, the tasks we design, and the discussions we prompt can refocus students on discovering such relationships themselves.

Emphasis on discovering relationships between numbers has been a positive change in my curriculum, but it is only a natural result of slowing the pace at which I move my students through my scope and sequence. At first, I was concerned about how I would make time to teach all that I used to teach. However, *Curriculum Focal Points* was written to encourage a critical look at curriculum revision, and studies such as TIMSS reveal that countries that outperform the United States in mathematics have a narrower curriculum focus by grade band. Schmidt (2004) identifies the three mathematics topics for grades 1 and 2 in such high achieving countries: whole-number meaning, whole-number operations, and measurement units—the same three focus areas that NCTM suggests for second grade.

One Japanese class spent eleven days building a thorough understanding of composing ten (Murata and Fuson 2006). This prolonged study is typical of the Japanese mathematics curriculum, which spends concentrated time periods on fewer topics, and the topics build on one another. Students in China also spend time composing and decomposing ten as part of the knowledge package for related concepts, such as subtraction with regrouping (Lipin 1999).

Although I was usually further along in developing scope and sequence at this same point in the academic year, I was nevertheless confident that the depth at which my students were developing number sense and efficient computation skills would pay dividends in other concept areas and in preparing them for third grade. In fact, as I was working through my computation unit, I noted that the number-sense relationships my students were building were the perfect precursor for our next unit on place value.


Making tens made sense to students, and they soon began doing it mentally.

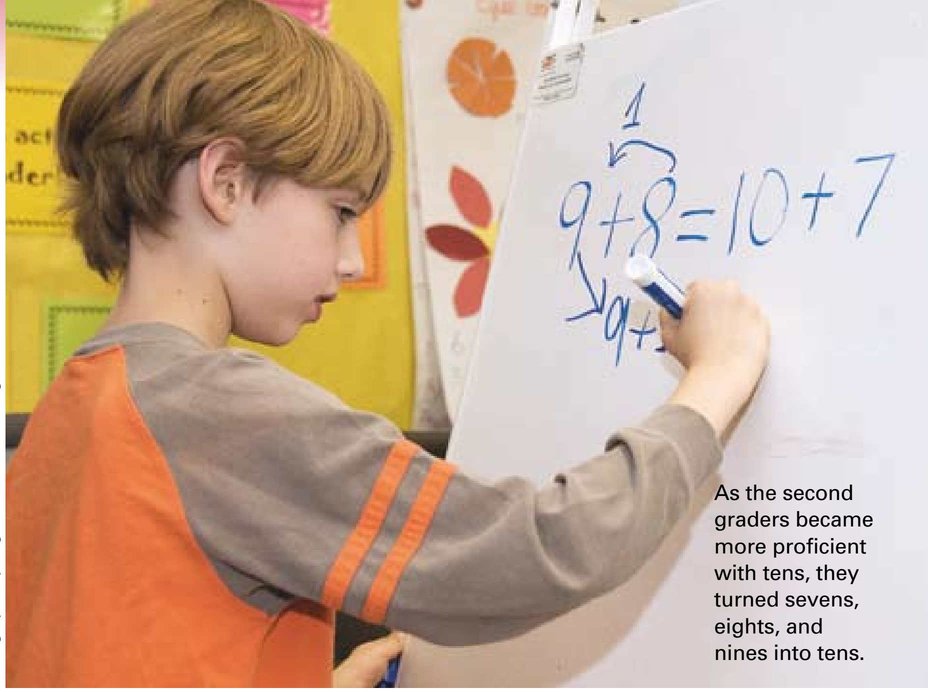
Students used a math balance beam as an effective tool to prove equivalency.

Place Value

Our work with the concept of ten influenced our approach to place value. The first task my students had to perform was how to count a large group of beans. They quickly suggested groups of ten and then used this method to name the number of beans. They named seven groups of beans and two singles as seventy-two. Students later applied the concept of composing tens to help them find the other part of one hundred. The tens frames were used again to initially model this idea. For example, if they modeled thirty-six and wanted to find out how many more were needed to make one hundred, they bridged to forty by adding four to thirty-six, and then they applied their making-tens strategy. So, if 4 + 6 = 10, then 4 tens + 6 tens = 100. They were able to deduce that 64 is the other part of 100. This model also helped them when thinking about making change for one dollar.

My second graders were now confidently trading ten ones for a ten, ten tens for a hundred, and decomposing larger units for smaller ones. Julia announced, “Ten is just the magic number!” Many of them formed mental number lines and bridged through tens numbers to find the difference between two numbers.

My curriculum evolves each year, but publication of the Focal Points, my reading and implementing of Van de Walle’s work with tens frames, and the great interest in the mathematics curricula of countries that outperform the United States have had a positive effect on how I teach mathematics and shape my students’ understanding of number. The result is a more in-depth study of numbers, in particular, number relationships and composing and decomposing ten. So far, I have seen students’ efficiency with computation improve, understanding of place-value relationships increase, and mathematical confidence soar. The final results of slowing down my curriculum to allow students to become more confident and efficient with numbers will be revealed in third grade. Scope and sequence will be evaluated and revised on the basis of outcomes of refocusing on the Focal Points for second grade. Further evaluation at other grades can then be a goal. As my team teacher and Postlewait, Adams, and Shih (2003) suggest, if students do not get a chance to develop such important number relationships as the composition of ten in the early grades, when will they?



As the second graders became more proficient with tens, they turned sevens, eights, and nines into tens.

Bibliography

Economopoulos, Karen, Joan Akers, Doug Clements, Anne Goodrow, Jerrie Moffet, and Julie Sarama, eds. *Mathematical Thinking at Grade 2*. In Investigations in Number, Data, and Space series. Menlo Park, CA: Dale Seymour Publications, 1998.

Griffin, Sharon. "Teaching Number Sense." *Educational Leadership* 61 (February 2004): 39–42.

Lipin, Ma. *Knowing and Teaching Elementary Mathematics: Teachers' Understanding of Fundamental Mathematics in China and the United States*. Mahwah, NJ: Lawrence Erlbaum Associates, 1999.

Murata, Aki, and Karen Fuson. "Teaching as Assisting Individual Constructive Paths within an Interdependent Class Learning Zone: Japanese First Graders Learning to Add Using 10." *Journal for Research in Mathematics Education* 37, no. 5 (November 2006): 421–56.

National Council of Teachers of Mathematics (NCTM). *Curriculum Focal Poaints for Prekindergarten through Grade 8 Mathematics*. Reston, VA: NCTM, 2006.

O'Loughlin, Tricia. "Using Research to Develop Computational Fluency in Young Mathematicians." *Teaching Children Mathematics* 14 (October 2007): 132–38.

Postlewait, Kristian B., Michelle R. Adams, and Jeffrey C. Shih. "Promoting Meaningful Mastery of Addition and Subtraction." *Teaching Children Mathematics* 9 (February 2003): 354–57.

Schmidt, W. H., C. C. McKnight, and S. A. Raizen. *A Splintered Vision: An Investigation of U.S. Science and Mathematics Education*. Dordrecht, Netherlands: Kluwer, 1997.

Schmidt, W. H., *Educational Leadership* 61, no. 5 (2004): 6–11.

Thompson, Charles S., and John A. Van de Walle. "Let's Do It! The Power of 10." *Arithmetic Teacher* 32 (November 1984): 6–11.

Van de Walle, John A. *Elementary and Middle School Mathematics: Teaching Developmentally*. 6th ed. New York: Pearson Education, 2007. ▲