Helping Our Students Become Mathematical Thinkers

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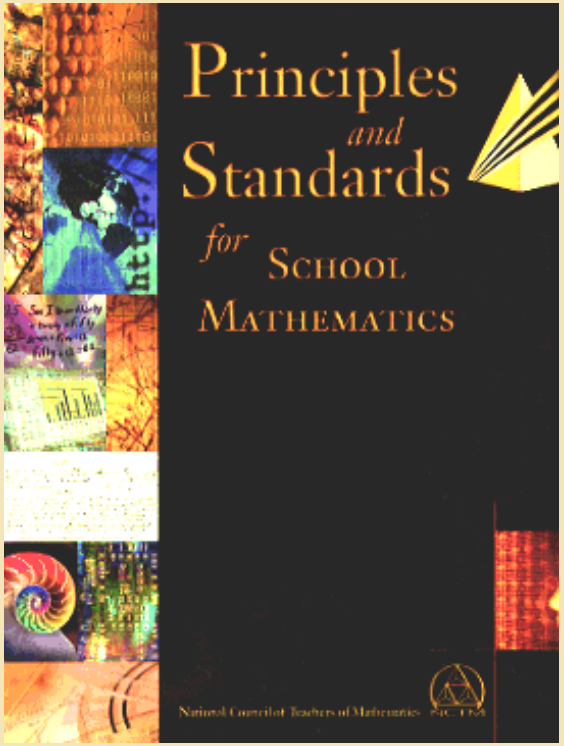
http://sites.jcu.edu/cmsett/serving-teachers/presentations/

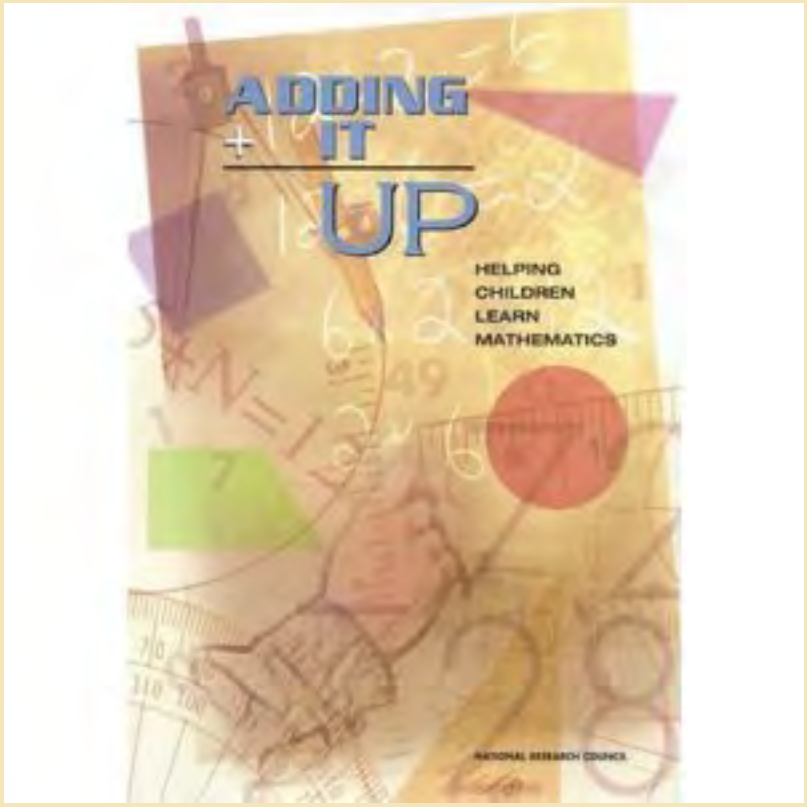
A Call for Coherence

Brief History

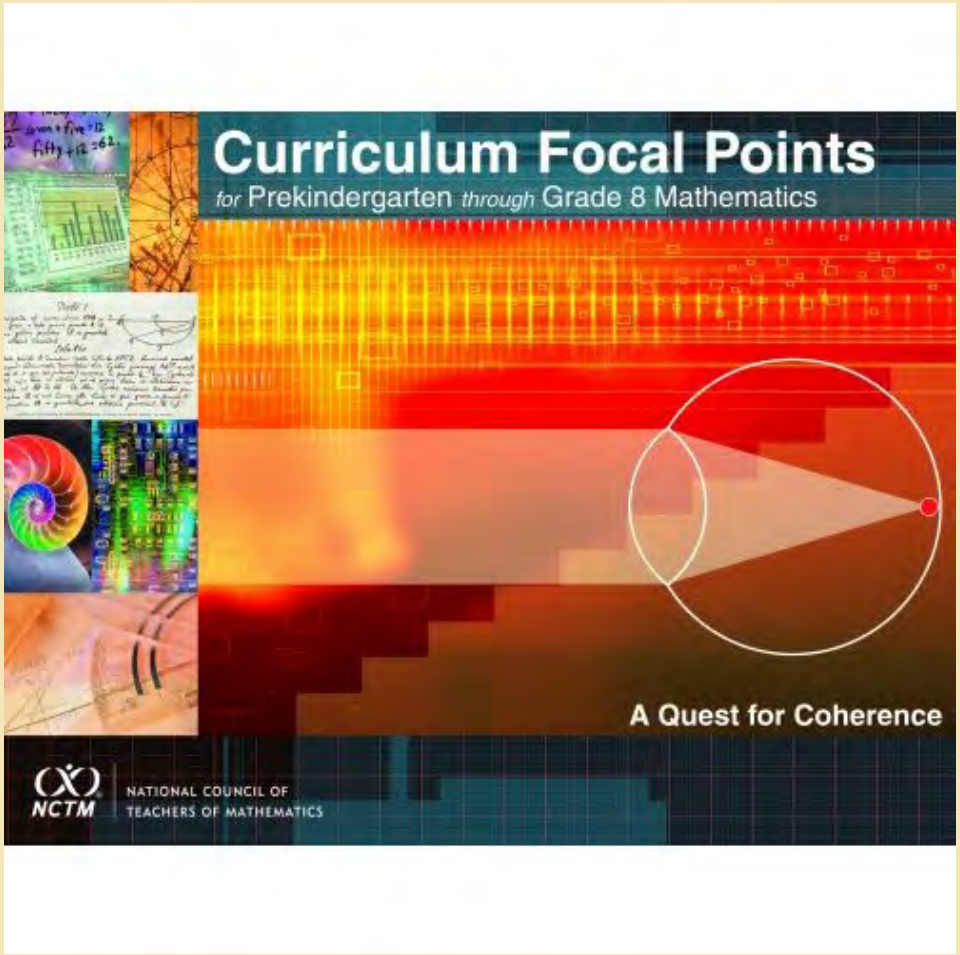
1989 NCTM Curriculum and Evaluation Standards



2000 NCTM Principles and Standards for School Mathematics



2001 National Research Council



2006 NCTM

Why you need to know....

Common Core Standards
www.corestandards.org

- Content Standards
  - Focus
  - Coherence
  - Clarity and specificity

- "Less is More"

Standards for Mathematical Practice

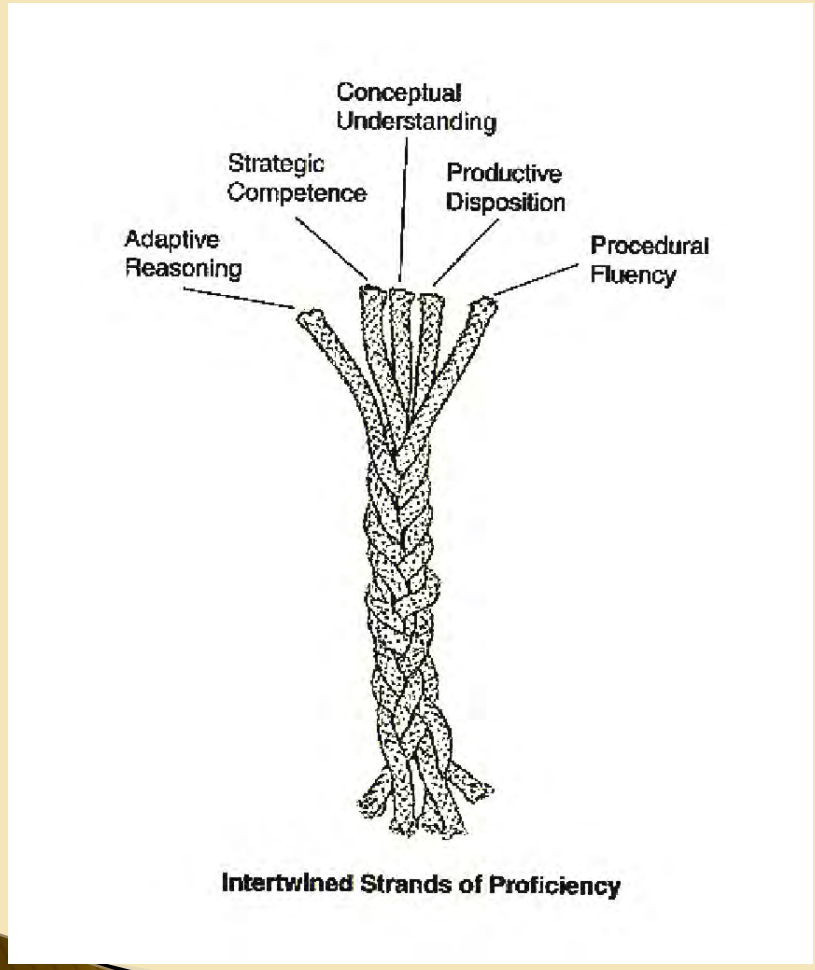
- For ALL students to become proficient in mathematics, they must internalize the eight mathematical practices as the means to learn, understand, and retain the content standards. The practices sustain mathematics as the content evolves. They define what is needed to be a quantitatively literate citizen.
  - Conference on Curriculum Design and Implementation Report 10/1-3/2010

NCTM Process Standards

- Connections
- Communication
- Problem Solving
- Reasoning and Proof
- Representation

- How do you currently address the Process Standards in your classroom?

National Research Council's Strands of Proficiency Adding It Up, 2001



- Adaptive Reasoning
- Strategic Competence
- Conceptual Understanding
- Productive Disposition
- Procedural Fluency
- www.nap.edu

Productive disposition –

- habitual inclination to see mathematics as
- sensible, useful, and worthwhile, coupled
- with a belief in diligence and one's own
- efficacy,

Never being of the mind to ask,
WHEN ARE WE EVER GOING TO USE THIS?!

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education.

8 CCSSM Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.

8 CCSSM Mathematical Practices

5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

I believe

We do not “teach” the practices, rather we design our instruction around tasks that give students the opportunity to use and develop the practices in learning and doing mathematics everyday, not only in mathematics class but in all aspects of their education and lives that include mathematical thinking.

1. Make sense of problems and persevere in solving them.

Students should be able to:

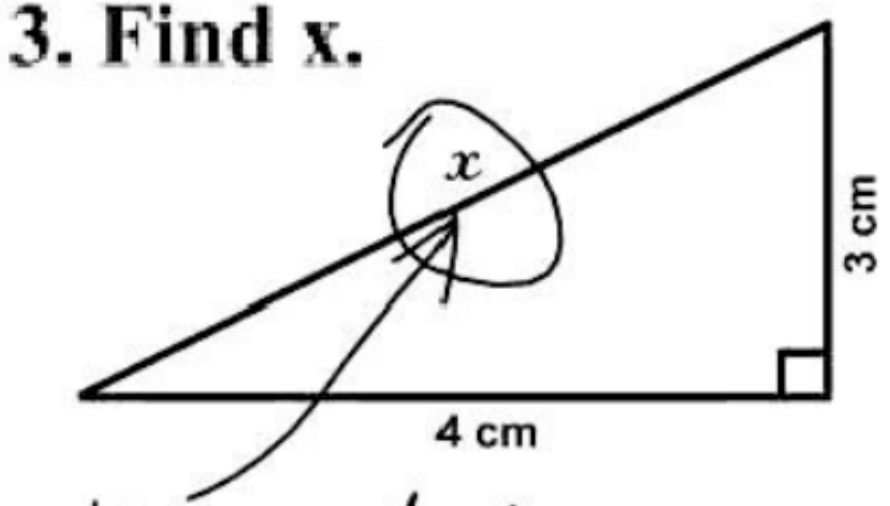
- Explain the meaning of the problem. May use concrete objects and/or pictorial representations.
- Develop a strategy for solving the problem.
- Identify the connections between two different approaches to a problem.
- Determine whether or not the solution makes sense.

1. Make sense of problems and persevere in solving them.

Students will need:

- Rich problems to consider.
- Time to reflect on their own thinking.
- Opportunities to dialogue with other students
- A safe environment to share their solutions with other students.

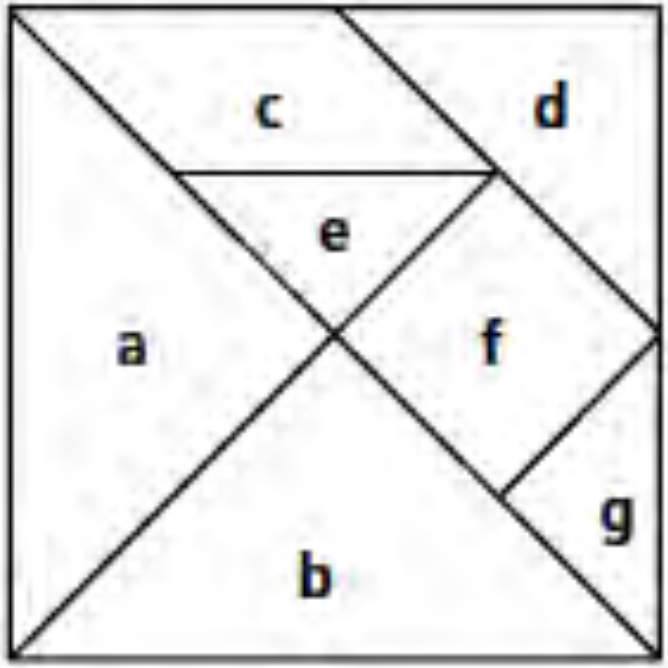
3. Find x.



Here it is

In this tangram puzzle, you can make many different shapes by putting the seven pieces together in different ways.

If the area of the entire puzzle is one square unit, find the area of each of the pieces.



2. Reason abstractly and quantitatively.

Students should be able to:

- De-contextualize – comprehend a given situation and represent it symbolically.
- Contextualize – consider the referents for the symbols they are working with.
- Understand the meaning of the quantities, not just how to compute them.

Contextualizing

- Place the decimal point

7.836 X 4.92 = 3855312

534.6 X 0.545 = 291357

51.1875 ÷ 1.05 = 4875

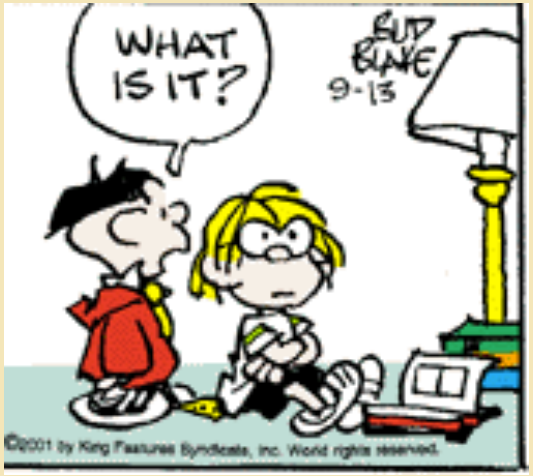
3.75 ÷ .05 = 750

Got Change?

Francine went to the store to buy some fruit. She can get 3 bananas for 50¢. She has lots of coins but no half dollars and no pennies. How many different ways could Francine make 50¢ to pay for the bananas?


CAN YOU ANSWER AN ARITHMETIC QUESTION FOR ME, JULIAN?
www.kingfeatures.com


SURE


WHAT IS IT?
BUD BLAKE
9-13
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3. Construct viable arguments and critique the reasoning of others.

Students should be able to:

- Make conjectures.
- Use counterexamples in their arguments.
- Justify their conclusions and explain them to others.
- Listen and/or read other's arguments and determine if they make sense.
- Ask questions to get clarification of an explanation.


100X WINS
10X 10X
BONUS
COLOSSAL
BAR
5X
X-TREME
5¢
10
1
150


VOLUME
5¢
HELP
CASH
$7.50
2800 x 5X
14000
CREDITS WON
LINE 8
PAYS 2000

"That's right, I've decided to give myself zero pay raise this year."

4. Model with mathematics.

Students should be able to:

- Apply the mathematics they know to solve everyday problems.
- Use equations, graphs, tables, diagrams, etc., to show the mathematical relationships in their model.
- Think about whether the model they have created makes sense and modify it if necessary.

Perplexing Population…

- Before coming to Philadelphia, we visited Lancaster, PA which is known for its Amish population. Our guide said that about every ten years the number of Amish people of the area doubles. If there were 10,000 Amish in Lancaster in 1980, how many Amish will there be in 2020? Draw a graph which represents the increase in population from 1980 to 2020.

5. Use appropriate tools strategically.

Students should be able to:

- Consider which available tools (calculator, ruler, concrete objects…) they might use when solving a problem.
- Recognize the strengths and limitations of the tools they are using.
- Identify additional external resources, such as a website.

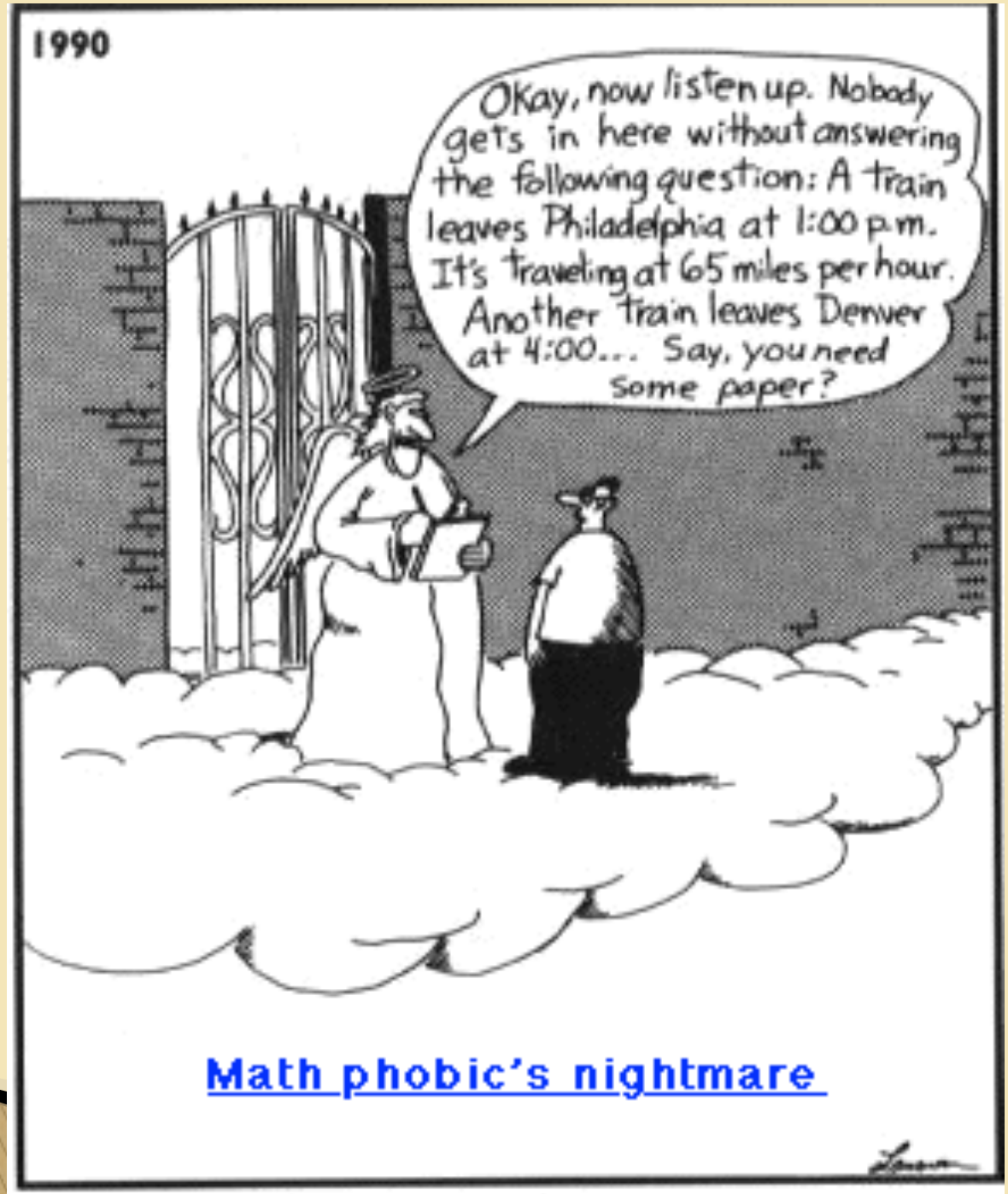
The Calculator Conundrum

62,327 ÷ 39

15 ÷ .25

Deal or No Deal

- Marty offers his parents a new deal for his allowance. Rather than getting $5 a week, he suggests they give him 1¢ for the first day, 2¢ for the second day, 4¢ for the third day and so on for the entire month of February. Should Marty's parents accept his deal?



Math phobic's nightmare

6. Attend to precision.

Students should be able to:

- Communicate precisely to others.
- Use clear definitions in discussion.
- Explain the meaning of the symbols they choose.
- Specify units of measure and label axes.
- Calculate accurately and efficiently.

- My house number is a three digit number. Each digit is different. It is an even number. It is divisible by 3. It is the closest number to 600 that fits these clues. What is my house number?

7. Look for and make use of structure.

Students should be able to:

- Look closely to identify a pattern or structure.
- Step back for an overview and shift perspective.
- See complicated things as single objects or as being composed of several objects.


PEANUTS
by Charles Schulz
"7+3=10"...THAT'S AN EASY ONE, MARCIE...
© 1961 United Feature Syndicate, Inc.
ANYTHING WITH A "3" IS EASY BECAUSE YOU JUST TAKE THE FIRST NUMBER AND THEN COUNT THE LITTLE POINTY THINGS ON THE "3," AND YOU HAVE THE ANSWER!
WHAT ABOUT "TWELVES," SIR?
NO ONE CAN BE EXPECTED TO ANSWER A PROBLEM WITH A "TWELVE" IN IT!
IF A PROBLEM HAS REALLY BIG NUMBERS IN IT, THE ANSWER IS ALWAYS "ONE MILLION"!
MATH IS LIKE LEARNING A FOREIGN LANGUAGE, MARCIE...NO MATTER WHAT YOU SAY, IT'S GOING TO BE WRONG ANYWAY!
LET'S SEE..."NINE PLUS THREE"...I TAKE THE NINE AND COUNT THE LITTLE POINTY THINGS ON THE THREE...TEN, ELEVEN, TWELVE...THE ANSWER IS "TWELVE"...HA!!
SCHULZ

8. Look for and express regularity in repeated reasoning.

Students should be able to:

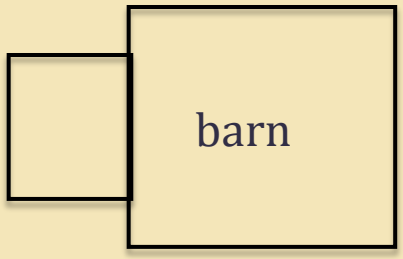
- Notice if calculations are repeated, and look for both general methods and for efficient ways to calculate.
- Maintain oversight of the process, while attending to the details.

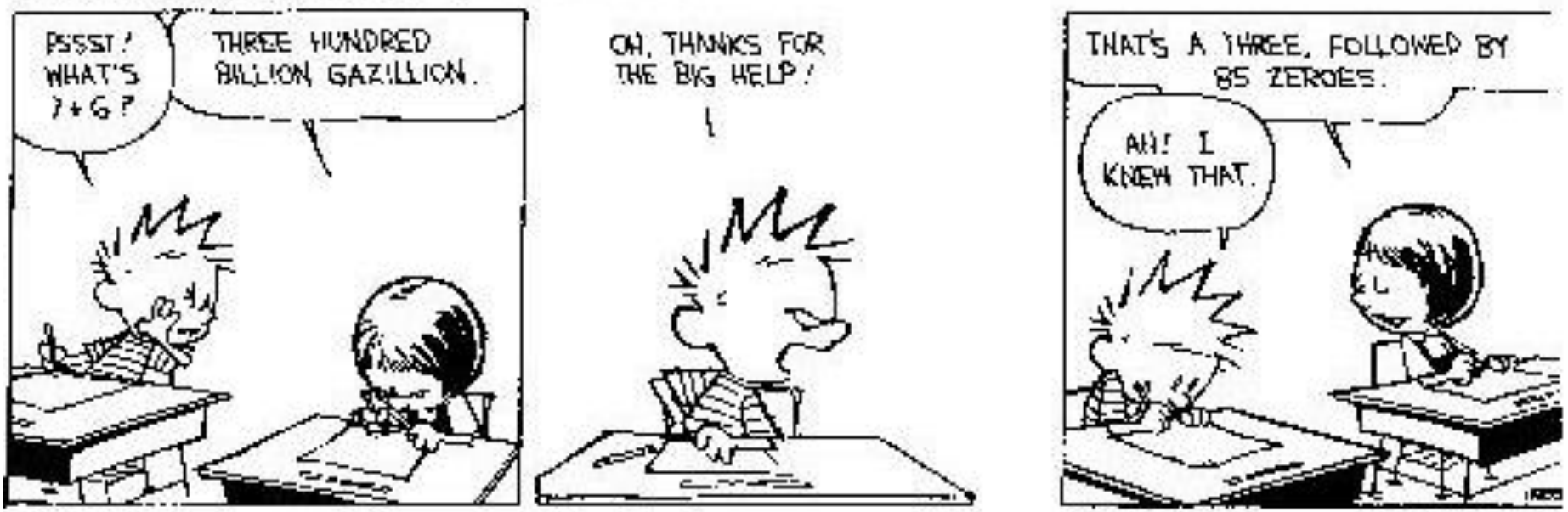
$$\begin{array}{r} 50 \\ \times\ 7 \\ \hline \end{array} \qquad \begin{array}{r} 50 \\ \times\ 70 \\ \hline \end{array}$$

$$8000 \times 5 =$$

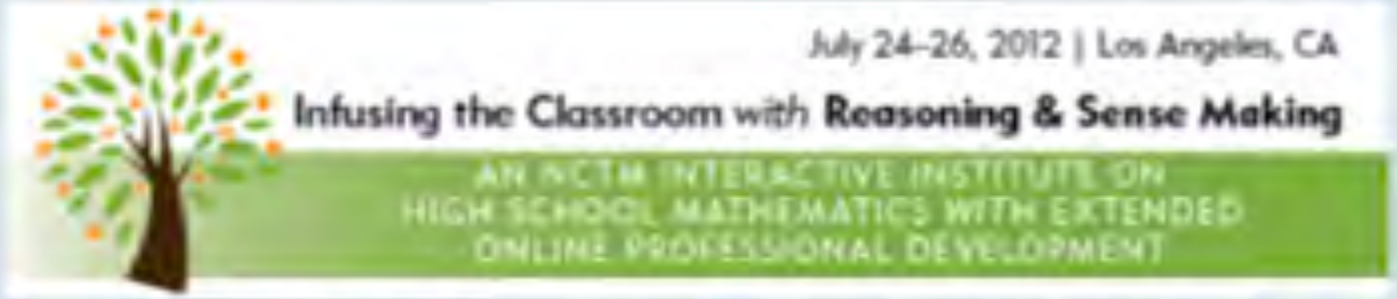
Moose's Playpen

- Tommy and Tessa want to fence in a play area for their dog, Moose. Moose is a big dog and they want to be sure to give him as much room as they can. They have 64 feet of fence. They can use up to 36 feet of the side of their barn as one side of the play area. The area must be rectangular and all of the sides must be a whole number of feet.
- Determine the best size for Moose's play area.




PSSST! WHAT'S 7+6?
THREE HUNDRED BILLION GAZILLION.
OH, THANKS FOR THE BIG HELP!
THAT'S A THREE, FOLLOWED BY 85 ZEROES.
AH! I KNEW THAT.

NCTM Interactive Institutes



- Reasoning and Sense Making for High School
- Key to Common Core mathematical practices
- 2½-day in-person professional development
- Full year of extended online professional development.
- University credit available
- www.nctm.org/reasoning

NCTM | NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

NCTM Interactive Institutes

- Algebra readiness for grades 3–8
- 2½-day in-person professional development
- Full year of extended online professional development
- University credit available
- www.nctm.org/algebra

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We can best close the achievement gap by eliminating the opportunity gap. If we, as mathematics teachers K-12, each make it our personal goal for every student to have the opportunity to learn mathematics in ways that promote the habits of mind espoused in the standards for mathematical practice, we will be successful in helping all students to be successful in learning and doing mathematics.

http://sites.jcu.edu/cmsett/serving-teachers/presentations