What's your math problem?

Linda Gojak
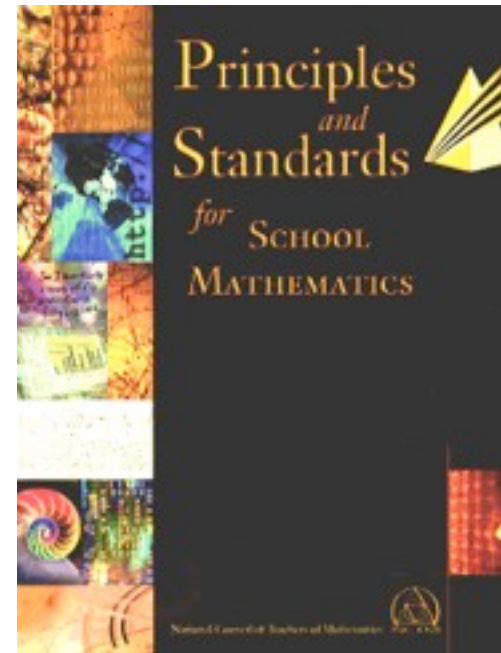
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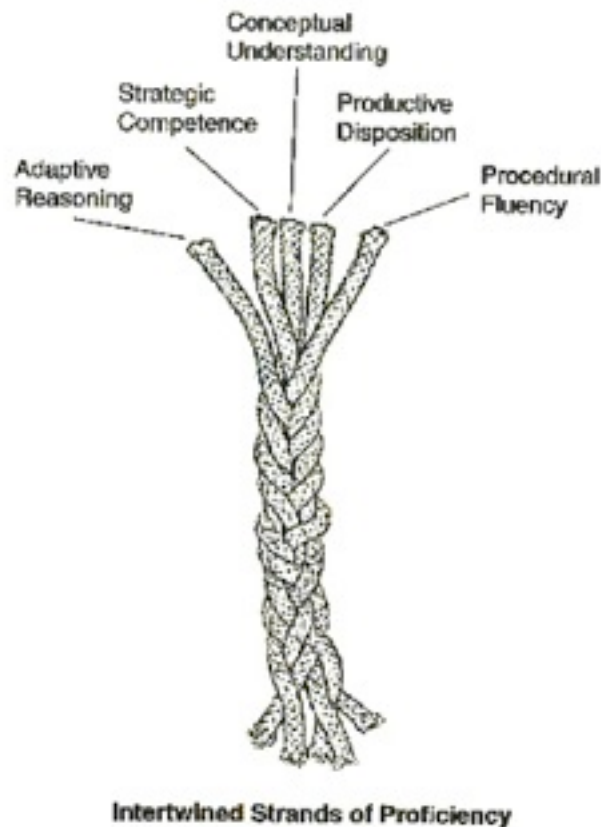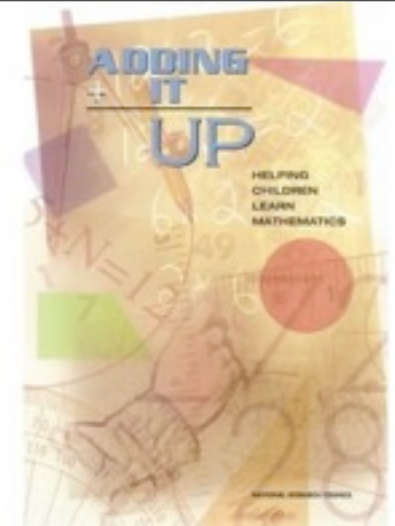
CMC North

December 2, 2011

NCTM Process Standards

–**Connections**

–**Communication**

–**Problem Solving**

–**Reasoning and Proof**

–**Representation**

Principles and Standards for School Mathematics

National Research Council's Strands of Proficiency

Intertwined Strands of Proficiency

- Adaptive Reasoning
- Strategic Competence
- Conceptual Understanding
- Productive Disposition
- Procedural Fluency

Problem Solving in the Common Core

Standards for Mathematical Practice

- Make sense of problems and persevere in solving them

Content Standards Grades 3-5

Solve problems involving the four operations, and identify and explain patterns in arithmetic. (Gr 3)

8. Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and

Problem solving develops the belief in students that they are capable of doing mathematics and the mathematics makes sense.

John VandeWalle

Thinking about Problem Solving

- What is problem solving?

- Where can you find good problems?

- Why teach problem solving?

- Who should solve problems?

- When can we teach problem solving?

- How do we teach problem solving?

- Why is problem solving important?
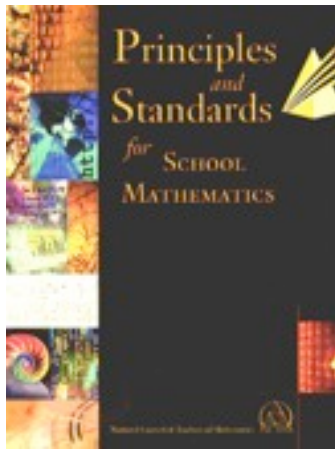
What is mathematical problem solving?

- Problem solving means engaging in a task for which the solution method is not known in advance.

- In order to find a solution, students must draw on their knowledge, and through this process, they will often develop new mathematical understandings.

- Solving problems is not only a goal of learning mathematics, but also a major means of doing so.

Principles and Standards for School Mathematics

Friday, December 2, 11

Using Strategies

Students use many strategies intuitively when they solve problems. However, gaining familiarity with a collection of strategies by seeing them modeled, and then trying to apply them, provides students with useful tools for tackling problems and broadens their problem solving abilities.

Marilyn Burns

Using Problem Solving Strategies

- A brief history

- Gives students entry into the problem

- Encourages process standards

- Helps students to learn mathematics

Using a strategy based approach

- LOOK FOR A PATTERN

- CONSTRUCT A TABLE

- ACCOUNT FOR ALL POSSIBLILITIES

- ACT IT OUT

- MAKE A MODEL

- WORK BACKWARDS

- GUESS AND CHECK

- MAKE A DRAWING

- MAKE A GRAPH

- SELECT APPROPRIATE NOTATION

- RESTATE THE PROBLEM

- IDENTIFY WANTED, NEEDED,GIVEN INFORMATION

- IDENTIFY A SUBGOAL

- SOLVE A SIMPLER PROBLEM

- CHANGE YOUR POINT OF VIEW

Getting Started Strategies

- Restate the problem in your own words

- Identify wanted, needed and given information

- Identify a subgoal

- Use appropriate notation

Solve this problem. Keep track of the order in which you use the clues.

Use the diagram to fill in the numbers using these clues:

- Each digit 1–9 is used once.
- Row 1 is half the total of Column 1.
- Row 1 contains the prime factorization of 30.
- There are no composite numbers in Column 3.
- The sums of Column 3 and Row 2 are the same.
- Column 2 contains even and consecutive numbers.
- All corner numbers are odd and consecutive.
- There is a square number in each column.

	Column 1	Column 2	Column 3	
				Row 1
				Row 2
				Row 3

Think about it…….

What mathematical ideas are explored in this problem?

What knowledge must students use to solve the problem?

What strategies did you use to solve the problem?

What strategies might your students use to solve the problem?

How do you think these strategies would help students?

How could you use the format of this problem to construct other problems?

13

What concerns do you have about incorporating problem solving into your mathematics instruction?

Marta was arranging some color tiles. She noticed a pattern as she laid out the tiles.   What would be the number of tiles in each of the next four designs? What would be the total number of tiles in the 20th design?

Design	1	2	3
Tiles	1	1 + 3 = 4	1 + 3 + 5 = 9
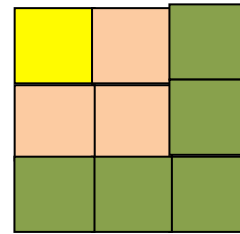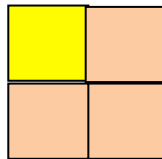
Think and Write...

1. What do I know? _____

2. What do I want to find out? _____

3. What extra information do I need? _____

4. Is there information I don't need? _____

5. Make an estimate or a prediction? _____

6. Which strategy/strategies can I use to solve this problem?

The L-E-S Instructional Model

- Launch
  - Setting the stage
  - Based on Polya's work

- Explore
  - Students use strategies to solve the problem

- Summarize
  - Students reflect on solutions and processes used

Most, if not all, important mathematics concepts and procedures can be best taught through problem solving. That is, tasks or problems can and should be posed that engage students in thinking about developing the important mathematics they need to learn.

NCTM 2000

Getting Organized Strategies

- **Look for a pattern**

- **Create a Table**

- **Create an Organized List**

- **Guess and Check**

Iggy and the Ice Cream Bandit

Iggy loves ice cream! His favorite flavor is carmel pecan swirl. His little sister, Izzy also loves ice cream. In fact she loves every flavor! Last week Iggy put a gallon of ice cream in the freezer. That night, Izzy ate half of the ice cream. The next night she ate half of what was left. This continued for a total of six nights. On the seventh day, Iggy decided to have some of his ice cream. Boy, was he surprised when he opened the container. How much ice cream was left for Iggy? How much ice cream did Izzy eat?

Iggy and the Ice Cream Bandit

What do I know? _____

What do I want to find out? _____

What additional information do I need? _____

Is there information I don't need? _____

Make an estimate or a prediction? _____

Which strategy/strategies can I use to solve this problem?

_____

"We believe that if we want students to understand mathematics, it is more helpful to think of understanding as something that results from solving problems, rather than something we can teach directly."
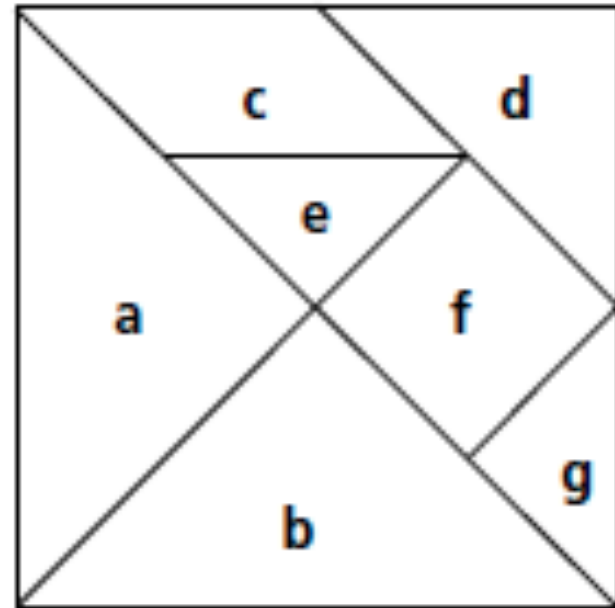
Heibert 1997

Visualizing Strategies

- Make a model

- Draw a picture or diagram

- Act it Out

- Create or use a graph

Tangrams

In a tangram puzzle, you can make many different shapes by putting the seven pieces together in different ways.

If the area of the large square is 1, find the area of each tangram piece.

The Handshake Problem

You are having a birthday party. When the first person arrives ,you shake her hand. As each new guest arrives, he or she shakes hands with everyone at the party. There are 10 people at the party. If each person shakes hands with everyone else exactly one time, how many handshakes will there be?

Additional Strategies

- Solve a simpler problem

- Account for all possibilities

- Work backwards

- Change your point of view

Gobs of Grapes

Mom just got home from the store with a big bunch of fresh juicy grapes. My brother came home from football practice and took ½ the grapes. My sister came home from her piano lesson and took ½ of what was left. When my dad got home from work he took ½ of the remaining grapes. By the time I finished my homework, there were only 6 grapes left. How many grapes were in the bunch that Mom bought?

Change your point of view….

Complete each of the following sequences.

S, M, T, W, T, _____, _____

O, T, T, F, F ____, _____, _____

F, S, T, F, F, S, S, _____, _____, _____

M, V, E, M, J, S, U, _____, _____

A farmer, a fox, a chicken, and a bag of corn must safely cross the river in a very small boat. The farmer may only take one thing at a time in the boat.

She cannot leave the fox and the chicken together on either side of the river, because the fox will eat the chicken. And, she cannot leave the chicken alone with the bag of corn, because the chicken will eat the corn. How can the farmer get everything across the river without anything being eaten?